

Light-Front Nuclear Physics: Mean Field Theory for Finite Nuclei

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Abstract

A light-front treatment for finite nuclei is developed from a relativistic effective Lagrangian (QHD1) involving nucleons, scalar mesons and vector mesons. We show that the necessary variational principle is a constrained one which fixes the expectation value of the total momentum operator P^+ to be the same as that for P^- . This is the same as minimizing the sum of the total momentum operators: $P^- + P^+$. We obtain a new light-front version of the equation that defines the single nucleon modes. The solutions of this equation are approximately a non-trivial phase factor times certain solutions of the usual equal-time Dirac equation. The ground state wave function is treated as a meson-nucleon Fock state, and the meson fields are treated as expectation values of field operators in that ground state. The resulting equations for these expectation values are shown to be closely related to the usual meson field equations. A new numerical technique to solve the self-consistent field equations is introduced and applied to ^{16}O and ^{40}Ca . The

computed binding energies are essentially the same as for the usual equal-time theory. The nucleon plus momentum distribution (probability for a nucleon to have a given value of p^+) is obtained, and peaks for values of p^+ about seventy percent of the nucleon mass. The mesonic component of the ground state wave function is used to determine the scalar and vector meson momentum distribution functions, with a result that the vector mesons carry about thirty percent of the nuclear plus-momentum. The vector meson momentum distribution becomes more concentrated at $p^+ = 0$ as A increases.

I. INTRODUCTION

The purpose of this paper is to derive the light-front formalism necessary to compute the properties of finite nuclei. Nuclear properties are very well handled within the existing conventional nuclear theory, so it behooves us to explain why we are embarking on this project. Our motivation is that understanding experiments involving high energy nuclear reactions seems to require that light-front dynamics and light cone variables be used. Consider the EMC experiment [1], which showed that there is a significant difference between the parton distributions of free nucleons and nucleons in a nucleus. This difference can be interpreted as a shift in the momentum distribution of valence quarks towards smaller values of the Bjorken variable x_{Bj} . The Bjorken variable is a ratio of the plus-momentum $k^+ = k^0 + k^3$ of a quark to that of the target. Thus light cone variables are relevant. If one uses k^+ as a momentum variable the corresponding canonical spatial variable is $x^- = x^0 - x^3$ and the time variable is $x^+ = x^0 + x^3$.

It is important to realize that the use of light-front dynamics is not limited to quarks within the nucleon — it also applies to nucleons within the nucleus. This formalism is useful whenever the momentum of initial or final state nucleons is large compared to their mass [2]. In particular, it can be used for $(e, e'p)$ and $(p, 2p)$ reactions. If one uses light-front variables for nucleons in a nucleus, it is also necessary to maintain consistency with the information derived previously using conventional nuclear dynamics. This provides the technical challenge which we address in the present manuscript.

The conventional equal-time approach to nuclear structure physics provides an excellent framework, so it is worthwhile to introduce the light-front variables and describe the expected advantages in a general way. The use of the light-cone variables can be obtained using a simple argument based on kinematics [2]. Suppose the virtual photon that is absorbed by a fermion at a space-time point (z_1, t_1) . The fermion then starts to move at high momentum and nearly the speed of light and emits the photon at another space time point (z_2, t_2) . In between the two times, the wave function of the entire system has undergone a time evolution given by the complicated operator $e^{-iH(t_2-t_1)}$. But we have $z_1 + ct_1 = z_2 + ct_2$, if the z -axis is opposite to the direction of the virtual photon. The two scattering events occur at different times, but at the same value of $x^+ = z + ct$. Thus if we use x^+ as a time variable, no time evolution factor appears. The net result is that the cross section involves light-like correlation functions which involve field operators evaluated at the same light-front time: $x^+ = 0$ (see for example the reviews [3,4]). Thus it is a specific and general feature of the light-front wave approach that knowing only the ground state wave function is sufficient

for computing the distribution functions.

Let us review the salient features of the basic idea that using the light-front approach leads to a simplified treatment. To be specific, consider high energy electron scattering from nucleons in nuclei. The key ingredient in the light-front simplification is to realize the main difference between the two formalisms. In the equal-time formalism, sums over intermediate states are taken over eigenstates of the Hamiltonian, P^0 . The usual three momentum is conserved, but energy is not conserved in intermediate states. In the light-front approach one sums over eigenstates of the minus component of the total momentum operator. The value of P^- is not conserved in intermediate state sums, and the values of P^+ and P^\perp are conserved. This is especially convenient for high energy reactions, in which the plus-component is the largest component of the momentum for each projectile or ejectile.

The advantage of using P^- as an “energy” variable can be easily described. Let the four-momentum q of the exchanged virtual photon be given by $(\nu, 0, 0, -\sqrt{Q^2 + \nu^2})$, with $Q^2 = -q^2$, and Q^2 and ν^2 are both very large but Q^2/ν is finite (the Bjorken limit). In this case it is worthwhile to use the light-cone variables $q^\pm = q^0 \pm q^3$ in which $q^+ \approx Q^2/2\nu = Mx$, $q^- \approx 2\nu - Q^2/2\nu$, so that $q^- \gg q^+$. Here M is the mass of a nucleon and x is the Bjorken variable. We shall neglect q^- in comparison to q^+ , noting that corrections to this can be handled in a systematic fashion. Then the schematic form of the scattering cross section for $e + A \rightarrow e' + (A-1)_f + p$, where f represents the final nuclear eigenstate of P^- , and p the four-momentum of the final proton, is given by

$$d\sigma \sim \sum_f \int \frac{d^3 p_f}{E_f} \int d^4 p \delta(p^2 - M^2) \delta^{(4)}(q + p_i - p_f - p) |\langle p, f | J(q) | i \rangle|^2. \quad (1.1)$$

Here the operator $J(q)$ is a schematic representation of the electromagnetic current. Performing the four-dimensional integral over p leads to the expression

$$d\sigma \sim \sum_f \int \frac{d^2 p_f dp_f^+}{p_f^+} \delta((p_i - p_f + q)^2 - m^2) |\langle p, f | J(q) | i \rangle|^2. \quad (1.2)$$

The argument of the delta function $(p_i - p_f + q)^2 - M^2 \approx -Q^2 + 2q^-(p_i - p_f)^+$. Thus we see that p_f^- does not appear in the argument of the delta function, or anywhere else, so that we can replace the sum over intermediate states by unity. In the usual equal-time representation, one finds the argument of the delta function to be $-Q^2 + 2\nu(E_i - E_f)$. The energy of the final state appears, and one can not do the sum.

To proceed further in this schematic approach we take

$$J(q) = \int d^3 k b_{\mathbf{k}+\mathbf{q}}^\dagger b_{\mathbf{k}}, \quad (1.3)$$

where b is a nucleon destruction operator and $\mathbf{V} \equiv (V^+, \mathbf{V}_\perp)$. It is useful to define $\mathbf{p}_B \equiv \mathbf{p}_i - \mathbf{p}_f$ because

$$p_B^+ = Q^2/2\nu \equiv Mx, \quad (1.4)$$

as demanded by the delta function. Then one can re-express Eq. (1.2) as

$$d\sigma \sim d^2p_{B\perp} \langle i | b_{\mathbf{p}_B}^\dagger b_{\mathbf{p}_B} | i \rangle = d^2p_{B\perp} n(Mx, p_{B\perp}), \quad (1.5)$$

where $n(Mx, p_{B\perp})$ is the probability for a nucleon in the ground state to have a momentum $(Mx, p_{B\perp})$. Integration in Eq. (1.5) leads to

$$\sigma \sim \int d^2p_\perp n(Mx, p_\perp) \equiv f(Mx), \quad (1.6)$$

with $f(Mx)$ as the probability for a nucleon in the ground state to have a plus momentum of Mx .

The quantity $f(Mx)$ has been a widely used prescription UP for handling the light-front in a simple way. The variable Mx is replaced by $M - \varepsilon_\alpha + k^3$, in which the label α denotes a shell-model orbital ϕ_α of binding energy ε_α . Then

$$f_{UP} = \sum_\alpha n_\alpha \int d^2p_\perp \int dp^3 | \phi_\alpha(p^3, p_\perp) |^2 \delta(M - \varepsilon_\alpha + p^3 - Mx), \quad (1.7)$$

in which n_α is an occupation probability. The validity of this prescription, which rests on a reasonable assumption, is rather suspect because the variable $p^+ = Mx$ is a kinematic variable, unrelated to discrete eigenvalues of a wave equation. One of the main purposes of the present paper is to see if anything like this prescription emerges from our calculations. We shall see that Eq. (1.7) is not obtained, if a vector potential is a significant part of the nuclear mean field.

It is useful to discuss the relation with y -scaling [5]. The arguments that the cross section depends on a plus-momentum distribution are well known when used for quarks in a nucleon, but they also apply to nucleons in a nucleus [2]. Ji and Filippone [6] showed that the y scaling function $F(y)$ extracted in quasi-elastic electron scattering on nuclei is actually the light cone plus-momentum distribution function for nucleons in the nucleus. It is useful to use a relativistic form of the variable y [7] in which

$$y = -q^3 + \nu + E_s + M, \quad (1.8)$$

as both q^3 and ν are large in magnitude, and E_s is the single nucleon separation energy. But $x = \frac{(q^3)^2 - \nu^2}{2M\nu} \approx 1 + \frac{E_s - y}{M}$, so that

$$Mx = M + E_s - y \equiv M_A y_A. \quad (1.9)$$

Here y_A is a new y -scaling variable. This means that according to Eq. (1.4)

$$p_B^+ = M_A y_A / M \approx A y_A, \quad (1.10)$$

so that a measurement of σ determines the probability that the struck nucleon has a plus-momentum of $A y_A$. This probability also enters in convolution model calculations of nuclear deep inelastic scattering.

The use of light-front dynamics to compute nuclear wave functions should allow us to compute $F(y)$ from first principles. Furthermore, we claim that using light-front dynamics incorporates the experimentally relevant kinematics from the beginning, and therefore is the

most efficient way to compute the cross sections for nuclear deep inelastic scattering and nuclear quasi-elastic scattering.

It is worthwhile to review some of the features of the EMC effect [1,4]. The key experimental result is the suppression of the structure function for $x \sim 0.5$. This means that the valence quarks of bound nucleons carry less plus-momentum than those of free nucleons. One way to understand this result is to postulate that mesons carry a larger fraction of the plus-momentum in the nucleus than in free space. While such a model explains the shift in the valence distribution, one obtains at the same time a meson (i.e. anti-quark) distribution in the nucleus, which is strongly enhanced compared to free nucleons and which should be observable in Drell-Yan experiments [8]. However, no such enhancement has been observed experimentally [9], and the implications are analyzed in Ref. [10].

The use of light-front dynamics allows us to compute the necessary nuclear meson distribution functions using variables which are experimentally relevant. The need for a computation of such functions in a manner consistent with generally known properties of nuclei led one of us to attempt to construct a light front treatment of nuclear physics [11]. These calculations, using a Lagrangian in which Dirac nucleons are coupled to massive scalar and vector mesons [12], treated the example of infinite nuclear matter within the mean field approximation. In this case, the meson fields are constants in both space and time and the momentum distribution has support only at $k^+ = 0$. Such a distribution would not be accessible experimentally, so that the suppression of the plus-momentum of valence quarks would not imply the existence of a corresponding testable enhancement of anti-quarks. However, it is necessary to ask if the result is only a artifact of the infinite nuclear size and of the mean field approximation. The present paper is an attempt to handle finite-sized nuclei using light-front dynamics.

A. Recovery of rotational invariance

It is worthwhile to discuss, in a general way, how it is that we are able to find spectra which have the correct number of degenerate states. Let us imagine that we try to determine eigenstates of a LF Hamiltonian by means of a variational calculation. Simply minimizing the LF energy obviously leads to nonsensical results since the LF energy scales like the inverse of the LF momentum. Even if one has only a poor ansatz for the intrinsic wave function, one can easily reach zero energy by letting the overall momentum scale to infinity! However, this problem is avoided by performing a constrained variation, in which the total LF momentum is fixed by including a Lagrange multiplier term proportional to the total momentum in the LF Hamiltonian. Note that this is not a problem if one is able to use a Fock space basis in which the total plus and \perp momentum of each component are fixed. In calculations involving many particles, the Fock state approach cannot be used in practical calculations — instead one uses a mean field in which each particle moves in an “external” potential. In this case the total momentum is not fixed, and a Lagrange multiplier term needs to be included in order to avoid solutions with infinite LF momentum.

In order to fix this potential problem with “runaway solutions” ($P^+ \rightarrow \infty$) to variational calculations for LF Hamiltonians, any term proportional to P^+ would suffice. However, by setting the coefficient for the term proportional to P^+ equal to one, *i.e.* minimizing

$P^- + P^+$, one automatically guarantees that $P^+ = P^-$ (or $P^3 = 0$). The reason is that, using covariance, P^- has eigenvalues of the form $P_n^- = \frac{M_n^2 + P_\perp^2}{P^+}$, *i.e.* it scales like $1/P^+$. Therefore, when one minimizes $P^- + P^+$ with respect to P^+ , the minimum occurs for $P^+ = \sqrt{M_n^2 + P_\perp^2}$, which yields $P^- = \frac{M_n^2 + P_\perp^2}{P^+} = \sqrt{M_n^2 + P_\perp^2}$ as well. This “equipartition” between P^+ and P^- thus arises since the two operators scale in exactly opposite ways under longitudinal boosts. Note that this is quite analogous to the nonrelativistic harmonic oscillator where, under scale transformations, potential and kinetic energy scale in opposite ways, resulting in the equipartition between potential and kinetic energy.

The net result is that we minimize the sum of $P^+ + P^-$. The need to include the plus-momentum can also be seen in a simple example. Consider a nucleus of A nucleons of momentum $P_A^+ = M_A$, $\mathbf{P}_{A\perp} = 0$, which consists of a nucleon of momentum (p^+, \mathbf{p}_\perp) , and a residual $(A - 1)$ nucleon system which must have momentum $(P_A^+ - p^+, -\mathbf{p}_\perp)$. The kinetic energy K is given by the expression

$$K = \frac{p_\perp^2 + M^2}{p^+} + \frac{p_\perp^2 + M_{A-1}^2}{P_A^+ - p^+}. \quad (1.11)$$

In the second expression, one is tempted to neglect the term p^+ in comparison with $P_A^+ \approx M_A$. This would be a mistake. Instead make the expansion

$$\begin{aligned} K &\approx \frac{p_\perp^2 + M^2}{p^+} + \frac{M_{A-1}^2}{P_A^+} \left(1 + \frac{p^+}{P_A^+}\right) \\ &\approx \frac{p_\perp^2 + M^2}{p^+} + p^+ + M_{A-1}, \end{aligned} \quad (1.12)$$

because for large A , $M_{A-1}^2/P_A^2 \approx 1$. For free particles, of ordinary three momentum \mathbf{p} one has $E^2(p) = \mathbf{p}^2 + m^2$ and $p^+ = E(p) + p^3$, so that

$$K \approx \frac{(E^2(p) - (p^3)^2)}{E(p) + p^3} + E(p) + p^3 + M_{A-1} = 2E(p) + M_{A-1}. \quad (1.13)$$

We see that K depends only on the magnitude of a three-momentum and rotational invariance is restored. The physical mechanism of this restoration is the inclusion of the recoil kinetic energy of the residual nucleus.

B. Outline

The organization of the paper is as follows. The light-front quantization for our chosen Lagrangian is presented in Sec. II. This quantization is applied, along with a constrained minimization of the expectation value of P^- , to derive a light-front version of mean field theory in Sec. III. We obtain a new light version of the equation that defines the single nucleon modes. The solutions of this equation are approximately a non-trivial phase factor times the solutions of the usual equal-time ET Dirac equation. The consequences of this phase factor are discussed.

The meson fields are treated as expectation values of operators. The equations for these expectation values are closely related to the meson field equations appearing in the usual treatment of the Walecka model. However, the mesonic Fock space is accessible in our formalism. Our nucleon mode equation is simplified by the use of a two-component spinor formalism [13], and by an angular momentum reduction in Sec. IV. The numerical aspects are discussed in App. A. The binding energies, nucleon and meson distributions for ^{16}O and ^{40}Ca are presented in Sec. V. A concluding discussion appears in Sec. VI. Numerical details of how we evaluate the momentum distributions are given in App. B. A brief discussion of some of the results can be found in Ref. [14]. A related set of solutions of some toy model problems and a heuristic derivation of our nucleon mode equation will appear in a separate paper [15].

II. LIGHT-FRONT QUANTIZATION

We start with a model in which the nuclear constituents are nucleons ψ (or ψ'), scalar mesons ϕ and vector mesons V^μ . The Lagrangian \mathcal{L} is given by

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi \partial_\mu \phi - m_s^2 \phi^2) - \frac{1}{4}V^{\mu\nu}V_{\mu\nu} + \frac{1}{2}m_v^2 V^\mu V_\mu + \bar{\psi}' \left(\gamma^\mu \left(\frac{i}{2} \overleftrightarrow{\partial}_\mu - g_v V_\mu \right) - M - g_s \phi \right) \psi', \quad (2.1)$$

where the bare masses of the nucleon, scalar and vector mesons are given by M, m_s, m_v , and $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. We ignore pions here.

The field equations are given by:

$$\gamma \cdot (i\partial - g_v V) \psi' = (M + g_s \phi) \psi', \quad (2.2)$$

$$\partial_\mu V^{\mu\nu} + m_v^2 V^\nu = g_v \bar{\psi}' \gamma^\nu \psi' \quad (2.3)$$

$$\partial_\mu \partial^\mu \phi + m_s^2 \phi = -g_s \bar{\psi}' \psi'. \quad (2.4)$$

The next step is obtain the light-front Hamiltonian (P^-) [16] as a sum of a free, non-interacting and a set of terms containing all of the interactions. This is accomplished by separating the independent and dependent degrees of freedom in the usual way [17,3] and then using the energy momentum tensor. Consider the nucleons. Although described by four-component spinors, these fields have only two independent degrees of freedom. The light-front formalism allows a convenient separation of dependent and independent variables via the projection operators $\Lambda_\pm \equiv \frac{1}{2} \gamma^0 \gamma^\pm$ [18,19], with $\psi'_\pm \equiv \Lambda_\pm \psi'$. The independent fermion degree of freedom is chosen to be ψ'_+ , and one finds

$$\begin{aligned} (i\partial^- - g_v V^-) \psi'_+ &= (\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \mathbf{V}_\perp) + \beta(M + g_s \phi)) \psi'_- \\ (i\partial^+ - g_v V^+) \psi'_- &= (\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \mathbf{V}_\perp) + \beta(M + g_s \phi)) \psi'_+. \end{aligned} \quad (2.5)$$

The relation between ψ'_- and ψ'_+ is very complicated unless one may set the plus component of the vector field to zero [17]. This is a matter of a choice of gauge for QED and QCD, but the non-zero mass of the vector meson prevents such a choice here. Instead, one simplifies the equation for ψ'_- by [18,20] transforming the fermion field according to

$$\psi' = e^{-ig_v \Lambda(x)} \psi, \quad \partial^+ \Lambda = V^+. \quad (2.6)$$

This transformation leads to the replacement of Eq. (2.5) by

$$\begin{aligned} (i\partial^- - g_v \bar{V}^-) \psi_+ &= (\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi)) \psi_- \\ i\partial^+ \psi_- &= (\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi)) \psi_+, \end{aligned} \quad (2.7)$$

where

$$\partial^+ \bar{V}^\mu = \partial^+ V^\mu - \partial^\mu V^+. \quad (2.8)$$

Note that while it is \bar{V}^μ that enters in the nucleon field equations, it is V^μ that enters in the meson field equations.

The scalar field can be expressed in terms of creation and destruction operators:

$$\phi(x) = \int \frac{d^2 k_\perp dk^+ \theta(k^+)}{(2\pi)^{3/2} \sqrt{2k^+}} \left[a(\mathbf{k}) e^{-ik \cdot x} + a^\dagger(\mathbf{k}) e^{ik \cdot x} \right], \quad (2.9)$$

where

$$k \cdot x = \frac{1}{2}(k^+ x^-) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp \equiv \mathbf{k} \cdot \mathbf{x}, \quad (2.10)$$

and the fields and their derivatives with respect to x^+ are evaluated at $x^+ = 0$. This notation is used through out this work. The consequence is that the energy momentum tensor $T^{\mu\nu}$ does not depend on x^+ . In the above expansion (and in the expansions for any of our fields) the particles are on the mass-shell. Here $k^- = \frac{k_\perp^2 + m_s^2}{k^+}$. The theta function restricts k^+ to positive values. The commutation relations are

$$[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = \delta(\mathbf{k}_\perp - \mathbf{k}'_\perp) \delta(k^+ - k'^+), \quad (2.11)$$

with $[a(\mathbf{k}), a(\mathbf{k}')] = 0$. It is useful to define

$$\delta^{(2,+)}(\mathbf{k} - \mathbf{k}') \equiv \delta(\mathbf{k}_\perp - \mathbf{k}'_\perp) \delta(k^+ - k'^+). \quad (2.12)$$

The expression for the vector meson field operator is

$$V^\mu(x) = \int \frac{d^2 k_\perp dk^+ \theta(k^+)}{(2\pi)^{3/2} \sqrt{2k^+}} \sum_{\omega=1,3} \epsilon^\mu(\mathbf{k}, \omega) \left[a(\mathbf{k}, \omega) e^{-i\mathbf{k} \cdot \mathbf{x}} + a^\dagger(\mathbf{k}, \omega) e^{i\mathbf{k} \cdot \mathbf{x}} \right], \quad (2.13)$$

where the polarization vectors are the usual ones:

$$\begin{aligned} k^\mu \epsilon_\mu(\mathbf{k}, \omega) &= 0, \quad \epsilon^\mu(\mathbf{k}, \omega) \epsilon_\mu(\mathbf{k}, \omega') = -\delta_{\omega\omega'}, \\ \sum_{\omega=1,3} \epsilon^\mu(\mathbf{k}, \omega) \epsilon^\nu(\mathbf{k}, \omega) &= -(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_v^2}). \end{aligned} \quad (2.14)$$

Once again the four momenta are on-shell, with $k^- = \frac{k_\perp^2 + m_v^2}{k^+}$. The commutation relations are

$$[a(\mathbf{k}, \omega), a^\dagger(\mathbf{k}', \omega')] = \delta_{\omega\omega'} \delta^{(2,+)}(\mathbf{k} - \mathbf{k}'), \quad (2.15)$$

with $[a(\mathbf{k}, \omega), a(\mathbf{k}', \omega')] = 0$, and lead to commutation relations amongst the field operators that are the same as in Ref. [20].

We also need the eigenmode expansion for \bar{V}^μ . This is given by

$$\bar{V}^\mu(x) = \int \frac{d^2 k_\perp dk^+ \theta(k^+)}{(2\pi)^{3/2} \sqrt{2k^+}} \sum_{\omega=1,3} \bar{\epsilon}^\mu(\mathbf{k}, \omega) \left[a(\mathbf{k}, \omega) e^{-ik \cdot x} + a^\dagger(\mathbf{k}, \omega) e^{ik \cdot x} \right], \quad (2.16)$$

where, using Eqs.(2.8) and (2.13), the polarization vectors $\bar{\epsilon}^\mu(\mathbf{k}, \omega)$ are

$$\bar{\epsilon}^\mu(\mathbf{k}, \omega) = \epsilon^\mu(\mathbf{k}, \omega) - \frac{k^\mu}{k^+} \epsilon^+(\mathbf{k}, \omega). \quad (2.17)$$

Note that

$$\sum_{\omega=1,3} \bar{\epsilon}^\mu(\mathbf{k}, \omega) \bar{\epsilon}^\nu(\mathbf{k}, \omega) = -(g^{\mu\nu} - g^{+\mu} \frac{k^\nu}{k^+} - g^{+\nu} \frac{k^\mu}{k^+}). \quad (2.18)$$

Then we may construct the total four-momentum operator from

$$P^\mu = \frac{1}{2} \int dx^- d^2 x_\perp T^{+\mu}(x^+ = 0, x^-, \mathbf{x}_\perp), \quad (2.19)$$

with (as usual)

$$T^{\mu\nu} = -g^{\mu\nu} \mathcal{L} + \sum_r \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_r)} \partial^\nu \phi_r, \quad (2.20)$$

in which the degrees of freedom are labelled by ϕ_r . We need T^{++} and T^{+-} , which are

$$T^{++} = \partial^+ \phi \partial^+ \phi + V^{ik} V^{ik} + m_v^2 V^+ V^+ + 2\psi_+^\dagger i \partial^+ \psi_+, \quad (2.21)$$

and

$$\begin{aligned} T^{+-} = & \nabla_\perp \phi \cdot \nabla_\perp \phi + m_\phi^2 \phi^2 + \frac{1}{4} (V^{+-})^2 + \frac{1}{2} V^{kl} V^{kl} + m_v^2 V^k V^k \\ & + \bar{\psi} \left(\gamma_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}^-) + M + g_s \phi \right) \psi. \end{aligned} \quad (2.22)$$

This form is still not useful for calculations because the constrained field ψ_- contains interactions. We follow Refs. [18,21] in expressing ψ_- as a sum of terms, one ξ_- whose relation with ψ_+ is free of interactions, the other η_- containing the interactions. That is, rewrite the second of Eq. (2.7) as [13]

$$\begin{aligned} \xi_- &= \frac{1}{i\partial^+} (\boldsymbol{\alpha}_\perp \cdot \mathbf{p}_\perp + \beta M) \psi_+ \\ \eta_- &= \frac{1}{i\partial^+} (-\boldsymbol{\alpha}_\perp \cdot g_v \bar{\mathbf{V}}_\perp + \beta g_s \phi) \psi_+. \end{aligned} \quad (2.23)$$

Furthermore, define $\xi_+(x) \equiv \psi_+(x)$, so that

$$\psi(x) = \xi(x) + \eta_-(x), \quad (2.24)$$

where $\xi(x) \equiv \xi_-(x) + \xi_+(x)$. This separates the dependent and independent parts of ψ .

One needs to make a similar treatment for the vector meson fields. The operator V^{+-} , is determined by

$$V^{-+} = \frac{2}{\partial^+} [g_v J^+ - m_v^2 V^+ - \partial_i V^{i+}]. \quad (2.25)$$

Part of this operator is determined by a constraint equation, because the independent variables are V^+ and V^{i+} . To see this examine Eq (2.25), and make a definition

$$V^{+-} = v^{+-} + \omega^{+-}, \quad (2.26)$$

where

$$\omega^{+-} = -\frac{2}{\partial^+} J^+. \quad (2.27)$$

The sum of the last term of Eq (2.22) and the terms involving ω^{+-} is the interaction density. Then one may use Eqs. (2.22), (2.24), and (2.26) to rewrite the P^- as a sum of different terms, with

$$P_{0N}^- = \frac{1}{2} \int d^2 x_\perp dx^- \bar{\xi} (\gamma_\perp \cdot \mathbf{p}_\perp + M) \xi, \quad (2.28)$$

and the interactions

$$P_I^- = v_1 + v_2 + v_3, \quad (2.29)$$

with

$$v_1 = \int d^2 x_\perp dx^- \bar{\xi} (g_v \gamma \cdot \bar{V} + g_s \phi) \xi, \quad (2.30)$$

$$v_2 = \int d^2 x_\perp dx^- \bar{\xi} (-g_v \gamma \cdot \bar{V} + g_s \phi) \frac{\gamma^+}{2i\partial^+} (-g_v \gamma \cdot \bar{V} + g_s \phi) \xi, \quad (2.31)$$

and

$$\begin{aligned} v_3 = & \frac{g_v^2}{8} \int d^2 x_\perp dx^- \int dy_1^- \epsilon(x^- - y_1^-) \xi_+^\dagger(y_1^-, \mathbf{x}_\perp) \gamma^+ \xi_+(y_1^-, \mathbf{x}_\perp) \\ & \times \int dy_2^- \epsilon(x^- - y_2^-) \xi_+^\dagger(y_2^-, \mathbf{x}_\perp) \gamma^+ \xi_+(y_2^-, \mathbf{x}_\perp), \end{aligned} \quad (2.32)$$

where $\epsilon(x) \equiv \theta(x) - \theta(-x)$. The term v_1 accounts the emission or absorption of a single vector or scalar meson. The term v_2 includes contact terms in which there is propagation of an instantaneous fermion. The term v_3 accounts for the propagation of an instantaneous vector meson.

Our variational procedure will involve the independent fields ψ_+ , so we need to express the interactions $P_{0N}^-, v_{1,2}$ in terms of ξ_+ . A bit of Dirac algebra shows that

$$\begin{aligned}
P_N^- &\equiv P_{0N}^- + v_1 + v_2 \\
&= \int d^2x_\perp \frac{dx^-}{2} \xi_+^\dagger \left[2g_v \bar{V}^- \right. \\
&\quad \left. + \left(\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi) \right) \frac{1}{i\partial^+} \left(\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi) \right) \right] \xi_+. \quad (2.33)
\end{aligned}$$

It is worthwhile to define the contributions to P^\pm arising from the mesonic terms as P_s^\pm and P_v^\pm . Then one may use Eqs. (2.22) and (2.21) along with the field expansions to obtain

$$\begin{aligned}
P_s^- &= \frac{1}{2} \int d^2x_\perp dx^- \left(\nabla_\perp \phi \cdot \nabla_\perp \phi + m_s^2 \phi^2 \right) \\
&= \int d^2k_\perp dk^+ \theta(k^+) a^\dagger(\mathbf{k}) a(\mathbf{k}) \frac{k_\perp^2 + m_s^2}{k^+}, \quad (2.34)
\end{aligned}$$

$$P_s^+ = \int d^2k_\perp dk^+ \theta(k^+) a^\dagger(\mathbf{k}) a(\mathbf{k}) k^+, \quad (2.35)$$

$$P_v^- = \sum_{\omega=1,3} \int d^2k_\perp dk^+ \theta(k^+) \frac{k_\perp^2 + m_v^2}{k^+} a^\dagger(\mathbf{k}, \omega) a(\mathbf{k}, \omega) + v_3, \quad (2.36)$$

and

$$P_v^+ = \sum_{\omega=1,3} \int d^2k_\perp dk^+ \theta(k^+) k^+ a^\dagger(\mathbf{k}, \omega) a(\mathbf{k}, \omega). \quad (2.37)$$

The term v_3 is the vector-meson instantaneous term, and we include it together with the purely mesonic contribution to P_v^- because it is cancelled by part of that contribution.

Thus, our result for the total minus-momentum operator is

$$P^- = P_N^- + P_s^- + P_v^-, \quad (2.38)$$

and for the plus-momentum

$$P^+ = P_N^+ + P_s^+ + P_v^+, \quad (2.39)$$

where from Eq. (2.21)

$$P_N^+ \equiv \int d^2x_\perp \frac{dx^-}{2} 2\xi_+^\dagger i\partial^+ \xi_+. \quad (2.40)$$

III. MEAN FIELD THEORY

The light-front Schroedinger equation for the complete nuclear ground-state wave function $|\Psi\rangle$ is

$$P^- |\Psi\rangle = M_A |\Psi\rangle. \quad (3.1)$$

We choose to work in the nuclear rest frame so that we also need

$$P^+ | \Psi \rangle = M_A | \Psi \rangle. \quad (3.2)$$

We want to use a variational principle. One might think that one may simply minimize the expectation value of P^- , but this makes no sense because $P^+P^- = M_A^2$ when acting on the wave function. One would get a zero of P^- for an infinite value of P^+ . As explained in the Introduction, one must minimize the expectation value of P^- subject to the condition that the expectation value of P^+ is equal to the expectation value of P^- . This is the same as minimizing the average of P^- and P^+ , which is the rest-frame energy of the entire system. To this end we define a light-front Hamiltonian

$$H_{LF} \equiv \frac{1}{2} (P^+ + P^-). \quad (3.3)$$

We stress that H_{LF} is not usual the Hamiltonian, because the light-front quantization is used to define all of the operators that enter.

The wave function $| \Psi \rangle$ consists of a Slater determinant of nucleon fields $| \Phi \rangle$ times a mesonic portion

$$| \Psi \rangle = | \Phi \rangle \otimes | \text{mesons} \rangle, \quad (3.4)$$

and the mean field approximation is characterized by the replacements

$$\begin{aligned} \phi &\rightarrow \langle \Psi | \phi | \Psi \rangle \\ V^\mu &\rightarrow \langle \Psi | V^\mu | \Psi \rangle. \end{aligned} \quad (3.5)$$

We shall derive the meson field equations, and then determine the nucleon modes using a variational principle.

A. Meson field equations

We shall go through the derivation of the equation for the expectation value of $\phi(x)$ in a detailed fashion. Consider the quantity $H_{LF}a(\mathbf{k}) | \Psi \rangle$, and use commutators to obtain

$$H_{LF}a(\mathbf{k}) | \Psi \rangle = [H_{LF}, a(\mathbf{k})] | \Psi \rangle + M_A a(\mathbf{k}) | \Psi \rangle. \quad (3.6)$$

The operators P_s^\pm of Eqs. (2.34) and (2.35) and the standard commutation relations allow one to obtain

$$[H_{LF}, a(\mathbf{k})] = -\frac{k_\perp^2 + k^{+2} + m_s^2}{2k^+} a(\mathbf{k}) + \frac{J(\mathbf{k})}{(2\pi)^{3/2} \sqrt{2k^+}}, \quad (3.7)$$

where $\frac{J(\mathbf{k})}{(2\pi)^{3/2} \sqrt{2k^+}}$ is the commutator of the interaction, Eq. (2.29), between the scalar meson and the nucleon:

$$\frac{J(\mathbf{k})}{(2\pi)^{3/2} \sqrt{2k^+}} = \frac{1}{2} [P_I^-, a((\mathbf{k}))]. \quad (3.8)$$

We use Eqs. (2.30)-(2.32), and take the commutator of the interactions v_i with $a(\mathbf{k})$. Then re-express the results in terms of ξ to obtain

$$J(\mathbf{k}) = -\frac{1}{2}g_s \int d^2x_\perp dx^- e^{i\mathbf{k}\cdot\mathbf{x}} \bar{\xi}(\mathbf{x}) \xi(\mathbf{x}). \quad (3.9)$$

Take the overlap of Eq. (3.6) with $\langle \Psi |$ to find

$$\langle \Psi | a(\mathbf{k}) | \Psi \rangle \frac{k_\perp^2 + k^{+2} + m_s^2}{2k^+} = \frac{\langle \Psi | J(\mathbf{k}) | \Psi \rangle}{(2\pi)^{3/2} \sqrt{2k^+}}. \quad (3.10)$$

Multiply the above Eq. (3.10) by a factor $\frac{\sqrt{2k^+}}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}}$. Then add the result of that operation to its complex conjugate. The integral of the resulting equation over all \mathbf{k}_\perp and positive values of k^+ and using the field expansion (2.9) leads to the result

$$\begin{aligned} & \left(-\nabla_\perp^2 - \left(2 \frac{\partial}{\partial x^-} \right)^2 + m_s^2 \right) \langle \Psi | \phi(x) | \Psi \rangle = \\ & \langle \Psi | \int \frac{d^2k_\perp dk^+ \theta(k^+)}{(2\pi)^3} \left(J(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}} + J^\dagger(\mathbf{k}) e^{+i\mathbf{k}\cdot\mathbf{x}} \right) | \Psi \rangle. \end{aligned} \quad (3.11)$$

The evaluation of the right-hand-side of Eq. (3.11) proceeds by using Eq. (3.9) and its complex conjugate. The combination of those two terms allows one to remove the factor $\theta(k^+)$ and obtain a delta function from the momentum integral. That $\frac{1}{2}k^+$ appears in the exponential leads to the removal of the factor $\frac{1}{2}$ of Eq. (3.9). One can also change variables using

$$z \equiv \frac{-x^-}{2}, \quad \mathbf{x} \equiv (z, \mathbf{x}_\perp). \quad (3.12)$$

The minus sign enters to remove the minus sign between the two terms of the factor $k \cdot x$ in Eq. (2.10). Then one may re-define the operator $-\nabla_\perp^2 - \left(2 \frac{\partial}{\partial x^-} \right)^2$ appearing in Eq. (3.11) as $-\nabla^2$. Note that we previously [22] obtained the above relation 3.12 simply by examining the space-time diagram for a static source (independent of x^0). The net result is that

$$\left(-\nabla^2 + m_s^2 \right) \langle \Psi | \phi(\mathbf{x}) | \Psi \rangle = -g_s \langle \Psi | \bar{\psi}(\mathbf{x}) \psi(\mathbf{x}) | \Psi \rangle, \quad (3.13)$$

which has the same form as the equation in the usual equal-time formulation. Note that the right hand side of Eq. (3.13) should be a function of $|\mathbf{x}|$ for the spherical nuclei of our present concern. Our formalism for the nucleon fields uses \mathbf{x}_\perp and x^- as independent variables, so that obtaining numerically scalar and vector nucleon densities that depend only $x_\perp^2 + (x^-/2)^2$ will provide a central, vital test of our procedures and mean field theory. Assuming for the moment that this occurs, the scalar field $\langle \Psi | \phi(\mathbf{x}) | \Psi \rangle$ will depend only $|\mathbf{x}|$ according to (3.13).

We stress that the use of Eq. (3.12) is merely a convenient way to simplify the calculation — using it allows us to treat the \perp and minus spatial variables on the same footing, and to maintain explicit rotational invariance. We will obtain the mesonic plus-momentum distributions from the ground state expectation value of different operators.

The procedure of Eqs. (3.6) to (3.13) can also be applied to the vector fields. The appearance of the barred vector potential makes it necessary to display certain steps. The starting point is to consider the expression $H_{LF}a(\mathbf{k}) | \Psi \rangle$ and the interaction

$$\frac{J(\mathbf{k}, \omega)}{(2\pi)^{3/2}\sqrt{2k^+}} = \frac{1}{2}[P_I^-, a(\mathbf{k}, \omega)]. \quad (3.14)$$

Using equations (2.30)-(2.32), taking the commutator of the interactions v_i with $a(\mathbf{k})$, and re-expressing the results in terms of ξ , leads to

$$J(\mathbf{k}, \omega) = -\frac{1}{2}g_v \int d^2x_\perp dx^- e^{ik \cdot x} \bar{\xi}(\mathbf{x}) \gamma \cdot \bar{\epsilon}(\mathbf{k}, \omega) \xi(\mathbf{x}). \quad (3.15)$$

This, along with the other terms in the expression for $H_{LF}a(\mathbf{k}, \omega) | \Psi \rangle$, allows us to obtain

$$\langle \Psi | a(\mathbf{k}, \omega) | \Psi \rangle \frac{k_\perp^2 + k^{+2} + m_v^2}{2k^+} = \frac{\langle \Psi | -\frac{1}{2}g_v \int d^2x_\perp dx^- e^{ik \cdot x} \bar{\xi}(\mathbf{x}) \gamma \cdot \bar{\epsilon}(\mathbf{k}, \omega) \xi(\mathbf{x}) | \Psi \rangle}{(2\pi)^{3/2}\sqrt{2k^+}}. \quad (3.16)$$

The field equation for the vector mesons \bar{V}^μ is obtained by multiplying the above by $\bar{\epsilon}^\mu(\mathbf{k}, \omega)$, summing over ω and performing standard manipulations. We need to know the quantity

$$X^\mu(\mathbf{k}) \equiv \sum_{\alpha, \omega} \gamma_\alpha \bar{\epsilon}^\alpha(\mathbf{k}, \omega) \bar{\epsilon}^\mu(\mathbf{k}, \omega) \quad (3.17)$$

The use of Eqs. (2.18) and (2.17) leads to

$$X^\mu(\mathbf{k}) = -\gamma^\mu + 2\delta(\mu, -)\frac{\gamma \cdot k}{k^+} + \frac{k^\mu}{k^+}\gamma^+. \quad (3.18)$$

One makes familiar manipulations to obtain the result

$$(-\nabla^2 + m_v^2) \langle \Psi | \bar{V}^\mu(\mathbf{x}) | \Psi \rangle = g_v \langle \Psi | \bar{\xi}(\mathbf{x}) \gamma^\mu \xi(\mathbf{x}) | \Psi \rangle + \Delta^\mu, \quad (3.19)$$

with

$$\Delta^\mu(\mathbf{x}) \equiv -\langle \Psi | \int \frac{d^3x'}{(2\pi)^3} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')} \bar{\xi}(\mathbf{x}') \left(\frac{k^\mu}{k^+} \gamma^+ + 2\delta(\mu, -)\frac{\gamma \cdot k}{k^+} \right) \xi(\mathbf{x}') | \Psi \rangle \quad (3.20)$$

Note that (as in the derivation given above for ϕ) the variable k^+ (confined to positive values) is replaced by the inclusion of the complex conjugate term ($a^\dagger(\mathbf{k}, \omega)$ by a variable k^3 which ranges from $-\infty$ to ∞ . We proceed by first assuming that

$$(-\nabla^2 + m_v^2) \langle \Psi | V^\mu(\mathbf{x}) | \Psi \rangle = g_v \langle \Psi | \bar{\xi}(\mathbf{x}) \gamma^\mu \xi(\mathbf{x}) | \Psi \rangle \equiv J^\mu, \quad (3.21)$$

which is to be verified by showing that Eqs. (3.19) and (3.21) are consistent with the defining relation (2.8) Taking the difference between Eqs. (3.19) and (3.21) and using the defining relation leads [23] to a consistency requirement

$$\partial^\mu J^+ = \partial^+ \Delta^\mu. \quad (3.22)$$

For $\mu \neq -$ the above relation is verified by integration by parts in the expression (3.20) for Δ^μ . If $\mu = -$, one may use the definition (2.8) and that $\langle \Psi | V^+ | \Psi \rangle$ does not depend on x^+ to see that

$$\partial^- \langle \Psi | \bar{V}^- | \Psi \rangle = \partial^- \langle \Psi | V^- | \Psi \rangle. \quad (3.23)$$

Thus the validity of Eq. (3.21) is established.

B. Nucleon single-particle wave functions

The mesonic field equations are given in the previous subsection. The equation for the nucleon modes are to be found using the procedure of minimizing $P^- + P^+$ with respect to the nucleon wave function, subject to the condition that the normalization of the independent fields remains fixed. The nucleon field operators enter only in the term $P_N^- + P_N^+$, so that it is useful to define

$$H_{LF} \equiv \frac{1}{2} (P_N^- + P_N^+). \quad (3.24)$$

The specific operator is obtained by using Eq. (2.33) to find

$$H_{LF} = \int \frac{dx^-}{2} d^2x_\perp \xi_+^\dagger \mathcal{H}_{LF} \xi_+, \quad (3.25)$$

where

$$2\mathcal{H}_{LF} \equiv i\partial^+ + 2g_v \bar{V}^- + \left(\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi) \right) \frac{1}{i\partial^+} \left(\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi) \right). \quad (3.26)$$

The potentials appearing in Eq. (3.26) are independent of x^+ . This implies some simplifications: $\partial^+ \bar{V}^- = \partial^+ V^- - \partial^- V^+ = \partial^+ V^-$, so that $\bar{V}^- = V^-$, and (for $i = 1, 2$) $\partial^+ \bar{V}^i = \partial^+ V^i - \partial^i V^+ = -\partial^i V^+$. Using the relation (2.6) we find that $\bar{V}^i = -\partial^i \Lambda$.

The Slater determinant $|\Phi\rangle$ is defined by allowing A nucleon states, denoted by the index α to be occupied. For our Slater determinant the constrained minimization is given by the equation

$$\delta \int d^2x_\perp \frac{dx^-}{2} \langle \alpha | \Lambda_+ \left(\mathcal{H}_{LF} - \frac{p_\alpha^-}{2} \right) \Lambda_+ | \alpha \rangle = 0, \quad (3.27)$$

where the quantities p_α^- are the Lagrange multiplication factors for each occupied orbital. The relation (3.27) leads immediately to our mode equation

$$p_\alpha^- \Lambda_+ | \alpha \rangle = \left(i\partial^+ + 2g_v \bar{V}^- \right) \Lambda_- | \alpha \rangle + \left(\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi) \right) \frac{1}{i\partial^+} \left(\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi) \right) \Lambda_+ | \alpha \rangle. \quad (3.28)$$

The operators $\boldsymbol{\alpha}$ and β have non-zero values when appearing between Λ_+ and Λ_- , but vanish when appearing between the two identical projection operators. Thus we may obtain $\Lambda_- | \alpha \rangle$ as

$$\Lambda_- | \alpha \rangle = \frac{1}{i\partial^+} \left(\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi) \right) \Lambda_+ | \alpha \rangle, \quad (3.29)$$

or

$$i\partial^+ \Lambda_- | \alpha \rangle = \left(\boldsymbol{\alpha}_\perp \cdot (\mathbf{p}_\perp - g_v \bar{\mathbf{V}}_\perp) + \beta(M + g_s \phi) \right) \Lambda_+ | \alpha \rangle. \quad (3.30)$$

One may use Eq. (3.30) to rewrite Eq. (3.28) as

$$\begin{aligned} p_{\alpha}^{-} \Lambda_{+} | \alpha \rangle &= \left(i \partial^{+} + 2g_v \bar{V}^{-} \right) \Lambda_{+} | \alpha \rangle \\ &+ \left(\boldsymbol{\alpha}_{\perp} \cdot (\mathbf{p}_{\perp} - g_v \bar{\mathbf{V}}_{\perp}) + \beta(M + g_s \phi) \right) \Lambda_{-} | \alpha \rangle . \end{aligned} \quad (3.31)$$

Equations (3.31) and (3.30) are the essential results of this section. We have obtained the light-front version of the Hartree equations.

C. Nuclear energy

There are contributions to the expectation value of $P^{-} + P^{+}$ from the nucleons, scalar mesons, and vector mesons. The nucleonic term is given from the expectation value of the nucleonic part H_{LF} (3.26). Taking the nuclear expectation value of H_{LF} leads to a sum of matrix elements in the occupied states $| \alpha \rangle$. The use of the wave equation (3.28) leads to the result

$$\langle \Psi | H_{LF} | \Psi \rangle = \sum_{\alpha}^{\text{occ}} \frac{p_{\alpha}^{-}}{2}, \quad (3.32)$$

in which the sum is over α includes only occupied states.

The contribution from the scalar mesons E_s is given from the scalar meson terms of Eqs. (2.34) and (2.35) by

$$E_s = \frac{1}{2} \int d^2 k_{\perp} dk^{+} \theta(k^{+}) \left[\frac{k_{\perp}^2 + m_s^2}{k^{+}} + k^{+} \right] \langle \Psi | a^{\dagger}(\mathbf{k}) a(\mathbf{k}) | \Psi \rangle. \quad (3.33)$$

In our mean field approximation

$$\langle \Psi | a^{\dagger}(\mathbf{k}) a(\mathbf{k}) | \Psi \rangle = | \langle \Psi | a(\mathbf{k}) | \Psi \rangle |^2, \quad (3.34)$$

with the matrix element already known from Eq. (3.10). Then straightforward calculation leads to the result

$$E_s = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\mathbf{k}^2 + m_s^2} | \langle \Psi | J(\mathbf{k}) | \Psi \rangle |^2, \quad (3.35)$$

where $J(\mathbf{k})$ is given by Eq. (3.9), the replacement (3.12) is used, and as above k^{+} is replaced by k^3 . The above expression is strikingly familiar — it is the result obtained in standard equal-time calculations.

The vector meson contribution to the energy E_v is defined as the one-half of the sum of the terms of P_v^{\pm} of Eqs. (2.36) and (2.37). The calculation of P_v^{\pm} is rather similar to the one just done for the scalar mesons. One uses the results (3.15), (3.16), and X^{μ} (3.18). The effects of the instantaneous term v_3 are cancelled by the non- γ^{μ} term of X^{μ} , so that we find

$$E_v = -\frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\mathbf{k}^2 + m_v^2} | J_v(\mathbf{k}) |^2, \quad (3.36)$$

where

$$J_v(\mathbf{k}) \equiv g_v \langle \Psi | \int d^3x e^{i\mathbf{k}\cdot\mathbf{x}} \psi^\dagger(\mathbf{x}) \psi(\mathbf{x}) | \Psi \rangle. \quad (3.37)$$

The nuclear mass M_A is then given by

$$M_A = \sum_{\alpha}^{\text{occ}} \frac{p_{\alpha}^-}{2} + E_s + E_v, \quad (3.38)$$

with expressions for each of the contributions given above.

D. Relation with the equal-time formulation

Our main results obtained using the mean field approximation and including the recoil of the $A - 1$ nuclear system are embodied in the equations (3.31) and (3.30). We solve these equations below using a mixed momentum-coordinate space procedure in which the wave functions are $\langle p^+, \mathbf{x}_{\perp} | \alpha \rangle = \psi_{\alpha}(p^+, \mathbf{x}_{\perp})$. The values of p^+ are greater than zero. Thus the so-called spectrum condition that positive energy particles have only positive plus-momenta is maintained in our mean field approximation.

An intermediate step is to make an approximation by using coordinate space techniques. Here one does not maintain the spectrum condition in an exact manner. Then one can show there is a very close relationship (but approximate) between our $\psi_{\alpha}(x^-, \mathbf{x}_{\perp})$ and the usual solutions to the Dirac equation obtained from the equal-time ET formulation.

To see this, let's first consider the case where there is no vector potential at all ($\bar{V}^{\mu} \rightarrow 0$). Then multiply Eq. (3.30) by γ^+ and Eq. (3.31) by γ^- . Use $\gamma^{\pm} \Lambda_{\mp} | \alpha \rangle = \gamma^{\pm} (\Lambda_+ + \Lambda_-) | \alpha \rangle = \gamma^{\pm} | \alpha \rangle$, and then add the two equations. This gives

$$\left(\gamma^0 p_{\alpha}^- - \gamma^3 (2p^+ - p_{\alpha}^-) \right) \psi_{\alpha}(x^-, \mathbf{x}_{\perp}) = 2 \left(\boldsymbol{\gamma}_{\perp} \cdot \mathbf{p}_{\perp} + M + g_s \phi(x^-, \mathbf{x}_{\perp}) \right) \psi_{\alpha}(x^-, \mathbf{x}_{\perp}). \quad (3.39)$$

Convert this to ordinary coordinates using $x^- = -2z$, so that $p^+ = i\partial^+ = 2i\frac{\partial}{\partial x^-} \rightarrow -i\frac{\partial}{\partial z}$. The operator p^+ acts as a p^3 operator, and the result (3.39) looks like the Dirac equation of the equal-time formulation, except for the offending term $-p_{\alpha}^-$ multiplying the γ^3 . This motivates us to look for a solution of the form $\psi_{\alpha}(z, \mathbf{x}_{\perp}) = f(z) \psi_{\alpha}^{\text{ET}}(z, \mathbf{x}_{\perp})$, in which $f(z)$ is chosen to remove the offending term. The notation ET refers to the usual equal-time solution, because we see that $\psi_{\alpha}^{\text{ET}}$ obeys the usual ET Dirac equation

$$\left(\gamma^0 \frac{p_{\alpha}^-}{2} - \boldsymbol{\gamma} \cdot \mathbf{p} - M - g_s \phi(z, \mathbf{x}_{\perp}) \right) \psi_{\alpha}^{\text{ET}}(z, \mathbf{x}_{\perp}) = 0, \quad (3.40)$$

provided

$$f(z) = e^{ip_{\alpha}^- z/2}, \quad (3.41)$$

so that

$$\psi_{\alpha}(z, \mathbf{x}_{\perp}) = e^{ip_{\alpha}^- z/2} \psi_{\alpha}^{\text{ET}}(z, \mathbf{x}_{\perp}). \quad (3.42)$$

The quantity of interest is $\psi_\alpha(p^+, \mathbf{x}_\perp)$ which is expressed as

$$\psi_\alpha(z, \mathbf{x}_\perp) \approx \frac{1}{\sqrt{2\pi}} \int_0^\infty dp^+ e^{ip^+ z} \psi_\alpha(p^+, \mathbf{x}_\perp). \quad (3.43)$$

The approximation is that the correct version of $\psi_\alpha(p^+, \mathbf{x}_\perp)$ will have no support for $p^+ < 0$, but the approximation (3.43) does. We can determine this support by examining the inverse Fourier transform. This gives

$$\psi_\alpha(p^+, \mathbf{x}_\perp) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty dz e^{-i(p^+ - p_\alpha^-/2)z} \psi_\alpha^{\text{ET}}(z, \mathbf{x}_\perp), \quad (3.44)$$

which is a Fourier transform of the equal-time Dirac wave function at a z -component of momentum $p^+ - p_\alpha^-/2$. This is not exactly equal to zero when p^+ is zero or negative, but it is very small because $p_\alpha^-/2$ includes the nucleon mass. The relationship between the term $p_\alpha^-/2$ and the binding energy of the level denoted by α is

$$p_\alpha^-/2 = M - \varepsilon_\alpha. \quad (3.45)$$

Thus the relation (3.44) is just the usual equal-time procedure equal-time prescription, represented by Eq. (1.7), of replacing the kinematic variable p^+ by the combination of dynamical and kinematic variables $M - \varepsilon_\alpha + p^3$ for the orbital α :

$$p^+ \rightarrow M - \varepsilon_\alpha + p^3. \quad (3.46)$$

However, the prescription (3.46) is dramatically changed when the vector potential is included. To see this, multiply Eq. (3.31) by γ^- and Eq. (3.30) by γ^+ . Then Eq. (3.39) becomes

$$\begin{aligned} (\gamma^0(p_\alpha^- - 2g_v V^0) - \gamma^3(2p^+ - p_\alpha^- + 2g_v V^0)) \psi_\alpha(x^-, \mathbf{x}_\perp) = \\ 2(\boldsymbol{\gamma}_\perp \cdot \mathbf{p}_\perp + M + g_s \phi) \psi_\alpha(x^-, \mathbf{x}_\perp), \end{aligned} \quad (3.47)$$

in which we used $\bar{V}^- = V^- = V^0$. We again wish to reduce the coefficient of the γ^3 term to $2p^+$. This can be done with a new version of the multiplier $f(z)$. We find that the light-front wave function is given by

$$\psi_\alpha(p^+, \mathbf{x}_\perp) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty dz e^{-i(p^+ - p_\alpha^-/2)z} e^{-ig_v \Lambda(z, \mathbf{x}_\perp)} \psi_\alpha^{\text{ET}}(z, \mathbf{x}_\perp), \quad (3.48)$$

where

$$\begin{aligned} \partial^+ \Lambda(x^-, \mathbf{x}_\perp) &= V^0(x^-, \mathbf{x}_\perp), \\ \Lambda(z, \mathbf{x}_\perp) &= \int_z^\infty dz' V^0(z', \mathbf{x}_\perp), \end{aligned} \quad (3.49)$$

and

$$\gamma^0(p_\alpha^- - g_v V^0) \psi_\alpha^{\text{ET}}(z, \mathbf{x}_\perp) = (\boldsymbol{\gamma} \cdot \mathbf{p} + M + g_s \phi) \psi_\alpha^{\text{ET}}(z, \mathbf{x}_\perp). \quad (3.50)$$

The relation (3.48) tells us that the influence of the vector potential is to remove plus-momentum from the nucleons. This removal and enhancement of the nuclear vector meson content is the most dramatic result we have.

How accurate is Eq. (3.48)? This can only be addressed by solving the problem in a manner which respects the spectrum condition. The results show an astonishing agreement between the eigenvalues of Eq. (3.31) and those of the equal-time Dirac equation. Thus it should be safe to use Eq. (3.48) for qualitative purposes.

IV. TECHNICAL ASPECTS

The solution of the nucleon and meson field equations are discussed. The reduction of Eqs. (3.31) and (3.30) to a two-dimensional matrix equation is presented here. The new numerical technique involving splines is elaborated in App. A.

The nucleon mode equation resulting from the minimization of $\frac{1}{2}(P^+ + P^-)$ is given by the coupled set of equations (3.31) and (3.30). The meson fields ϕ and V^\pm obey the equations

$$\left(-(\partial^+)^2 - \partial_\perp^2 + m_s^2\right) \phi(x^-, \mathbf{x}_\perp) = -g_s \sum_{\alpha}^{\text{occ}} \bar{\psi}_\alpha(x^-, \mathbf{x}_\perp) \psi_\alpha(x^-, \mathbf{x}_\perp), \quad (4.1)$$

$$\left(-(\partial^+)^2 - \partial_\perp^2 + m_v^2\right) V^\pm(x^-, \mathbf{x}_\perp) = g_v \sum_{\alpha}^{\text{occ}} \bar{\psi}_\alpha(x^-, \mathbf{x}_\perp) \gamma^\pm \psi_\alpha(x^-, \mathbf{x}_\perp), \quad (4.2)$$

in which $\psi_\alpha(x^-, \mathbf{x}_\perp) \equiv \langle x^-, \mathbf{x}_\perp | \alpha \rangle$. We use the Harindranath-Zhang [13] representation for the Dirac matrices α and β , which allows us to write Eqs. (3.31) and (3.30) in 2-component form. This representation can be obtained from the standard representation

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (4.3)$$

by the unitary transformation

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\sigma_3 \\ \sigma_3 & 1 \end{pmatrix}. \quad (4.4)$$

Hence $\psi \rightarrow U\psi$ and $\theta \rightarrow U\theta U^\dagger$, where θ is a Dirac matrix in the standard representation. In our representation, the matrices of interest are:

$$\begin{aligned} \Lambda^+ &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, & \Lambda^- &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \\ \beta &= \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, & \alpha_3 &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, & \alpha_\perp &= \begin{pmatrix} 0 & \sigma_\perp \\ \sigma_\perp & 0 \end{pmatrix}. \end{aligned} \quad (4.5)$$

The 4-component wavefunction $|\psi_\alpha\rangle$ may now be written in the form

$$\langle x^-, \mathbf{x}_\perp | \psi_\alpha \rangle = \begin{pmatrix} \langle x^-, \mathbf{x}_\perp | \psi_\alpha^- \rangle \\ \langle x^-, \mathbf{x}_\perp | \psi_\alpha^+ \rangle \end{pmatrix}, \quad (4.6)$$

in terms of the 2-component wavefunctions $|\psi_\alpha^+\rangle$ and $|\psi_\alpha^-\rangle$. Thus the 2-component form of Eqs. (3.31) and (3.30) is

$$(p_\alpha^- - 2g_v V^- - i\partial^+) |\psi_\alpha^+\rangle = (\boldsymbol{\sigma}_\perp \cdot (\mathbf{p}_\perp - g_v \boldsymbol{\partial}_\perp \Lambda) + \sigma_3 (M + g_s \phi)) |\psi_\alpha^-\rangle, \quad (4.7a)$$

$$i\partial^+ |\psi_\alpha^-\rangle = (\boldsymbol{\sigma}_\perp \cdot (\mathbf{p}_\perp - g_v \boldsymbol{\partial}_\perp \Lambda) + \sigma_3 (M + g_s \phi)) |\psi_\alpha^+\rangle. \quad (4.7b)$$

The scalar and vector densities are defined as

$$\rho^s \equiv \sum_{\alpha}^{\text{occ}} \bar{\psi}_{\alpha} \psi_{\alpha} = \sum_{\alpha}^{\text{occ}} \left(\bar{\psi}_{\alpha}^{+} \sigma_3 \psi_{\alpha}^{-} + \bar{\psi}_{\alpha}^{-} \sigma_3 \psi_{\alpha}^{+} \right), \quad (4.8)$$

$$\rho^{\pm} \equiv \sum_{\alpha}^{\text{occ}} \bar{\psi}_{\alpha} \gamma^{\pm} \psi_{\alpha} = \sum_{\alpha}^{\text{occ}} 2(\psi_{\alpha}^{\pm})^{\dagger} \psi_{\alpha}^{\pm}. \quad (4.9)$$

In the nuclear rest frame, $\rho^{+} = \rho^{-} = \rho^0$, where ρ^0 is the usual nucleon density. Hence $V_{\perp} = 0$ and $V^{-} = V^{+}$ in this frame.

A. Angular momentum

We can write

$$\boldsymbol{\sigma}_{\perp} \cdot \mathbf{p}_{\perp} = \sigma_{(+)} p_{(-)} + \sigma_{(-)} p_{(+)}, \quad (4.10)$$

where $\sigma_{(\pm)} = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$ and

$$p_{(\pm)} = p_1 \pm ip_2 = -ie^{\pm i\phi} \left(\frac{\partial}{\partial r} \pm \frac{1}{r} \frac{\partial}{\partial \phi} \right). \quad (4.11)$$

Here $r = |\mathbf{x}_{\perp}|$ and ϕ is the azimuthal angle, using cylindrical coordinates. For the nuclear physics problems of interest, we anticipate that there is an axis of azimuthal symmetry. Hence we can expand the 2-component wavefunctions in eigenstates of angular momentum J_z , with eigenvalue j_z :

$$\begin{aligned} \langle x^{-}, \mathbf{x}_{\perp} | \psi_{\alpha}^{\pm} \rangle &= i \langle x^{-}, r | u_{\alpha}^{\pm} \rangle e^{i(j_z - \frac{1}{2})\phi} \chi_{\frac{1}{2}} + \langle x^{-}, r | l_{\alpha}^{\pm} \rangle e^{i(j_z + \frac{1}{2})\phi} \chi_{-\frac{1}{2}} \\ &= \begin{pmatrix} i u_{\alpha}^{\pm}(x^{-}, r) e^{i(j_z - \frac{1}{2})\phi} \\ l_{\alpha}^{\pm}(x^{-}, r) e^{i(j_z + \frac{1}{2})\phi} \end{pmatrix}, \end{aligned} \quad (4.12)$$

where $\chi_{\frac{1}{2}}$ and $\chi_{-\frac{1}{2}}$ are the 2-component Pauli spinors.

The equations to be solved are then

$$(p_{\alpha}^{-} - 2g_v V^{+} - i\partial^{+}) u_{\alpha}^{+} = - \left(\frac{\partial}{\partial r} + \frac{j_z + \frac{1}{2}}{r} - ig_v \frac{\partial \Lambda}{\partial r} \right) l_{\alpha}^{-} + M^{*} u_{\alpha}^{-}, \quad (4.13a)$$

$$(p_{\alpha}^{-} - 2g_v V^{+} - i\partial^{+}) l_{\alpha}^{+} = \left(\frac{\partial}{\partial r} - \frac{j_z - \frac{1}{2}}{r} - ig_v \frac{\partial \Lambda}{\partial r} \right) u_{\alpha}^{-} - M^{*} l_{\alpha}^{-}, \quad (4.13b)$$

$$i\partial^{+} u_{\alpha}^{-} = - \left(\frac{\partial}{\partial r} + \frac{j_z + \frac{1}{2}}{r} - ig_v \frac{\partial \Lambda}{\partial r} \right) l_{\alpha}^{+} + M^{*} u_{\alpha}^{+}, \quad (4.13c)$$

$$i\partial^{+} l_{\alpha}^{-} = \left(\frac{\partial}{\partial r} - \frac{j_z - \frac{1}{2}}{r} - ig_v \frac{\partial \Lambda}{\partial r} \right) u_{\alpha}^{+} - M^{*} l_{\alpha}^{+}. \quad (4.13d)$$

The wavefunctions u_{α}^{\pm} and l_{α}^{\pm} , the nucleon effective mass $M^{*} = M + g_s \phi$, the vector potential V^{+} , and Λ are all functions of both x^{-} and r . Eqs. (4.13) have a manifest spin degeneracy

under $j_z \rightarrow -j_z$. Solutions with the same eigenvalue p_α^- are obtained with the corresponding replacement

$$\begin{pmatrix} u_\alpha^+ \\ l_\alpha^+ \end{pmatrix} \rightarrow \begin{pmatrix} l_\alpha^+ \\ u_\alpha^+ \end{pmatrix}, \quad \begin{pmatrix} u_\alpha^- \\ l_\alpha^- \end{pmatrix} \rightarrow - \begin{pmatrix} l_\alpha^- \\ u_\alpha^- \end{pmatrix}, \quad (4.14)$$

Combined with isospin symmetry, we therefore have a manifest fourfold degeneracy of each single particle state. The numerical solution to Eqs. (4.13) is discussed in App. A.

V. NUCLEAR BINDING ENERGIES

If these solution to Eqs. (3.31) and (3.30) are to have any relevance at all, they should respect rotational invariance. The success in achieving this is examined in Tables I and II, which give our results for the spectra of ^{16}O and ^{40}Ca , respectively. Scalar and vector meson parameters are taken from Horowitz and Serot [24], and we have assumed isospin symmetry. We see that the $J_z = \pm 1/2$ spectrum contains the eigenvalues of all states, since all states must have a $J_z = \pm 1/2$ component. Furthermore, the essential feature that the expected degeneracies among states with different values of J_z are reproduced numerically.

TABLES

TABLE I. Comparison of the single particle spectra of ^{16}O in the equal-time (ET) formalism ($E_\alpha - M$) with the light-front (LF) method ($p_\alpha^-/2 - M$). Units are in MeV.

ET		LF	
State α	$E_\alpha - M$	$J_z = \pm 1/2$	$J_z = \pm 3/2$
$0s_{1/2}$	-41.73	-41.73	
$0p_{3/2}$	-20.77	-20.79	-20.77
$0p_{1/2}$	-12.49	-12.51	

TABLE II. Comparison of the ET and LF single particle spectra of ^{40}Ca .

ET		LF		
State α	$E_\alpha - M$	$J_z = \pm 1/2$	$J_z = \pm 3/2$	$J_z = \pm 5/2$
$0s_{1/2}$	-55.40	-55.39		
$0p_{3/2}$	-38.90	-38.91	-38.90	
$0p_{1/2}$	-33.18	-33.18		
$0d_{5/2}$	-22.75	-22.76	-22.75	-22.74
$1s_{1/2}$	-14.39	-14.36		
$0d_{3/2}$	-13.87	-13.88	-13.89	

TABLE III. Total plus-momentum per nucleon for ^{16}O , ^{40}Ca , ^{80}Zr , and nuclear matter (NM) in MeV. No Coulomb interaction is included here.

Nucleus	P_N^+/A	P_s^+/A	P_v^+/A	P^+/A
^{16}O	704.7	6.4	221.8	932.9
^{40}Ca	672.6	4.7	253.3	930.6
^{80}Zr	655.2	3.6	270.2	929.0
NM	569.0	0.0	354.2	923.2

The results shown in Tables I–III are obtained using a basis of 20 splines, a box size of $2L = 24$ fm, and 24 Fourier components in the expansion of the wavefunction (see App. A). This value of L is large enough so that our results do not depend on it, and the number of terms in the expression for the density is enough to ensure that the densities are spherically symmetric. Another feature is that the spectrum with $p^+ > 0$ has no negative energy states, so that in using the LF method one is working in a basis of positive energy states only.

The values of $p_\alpha^-/2$ given in Tables I and II are essentially the same as the single particle energies E_α of the ET formalism, to within the expected numerical accuracy of our program. This equality is not mandated by spherical symmetry alone because the solutions in the equal-time framework have non-vanishing components with negative values of p^+ .

Table III gives the contributions to the total P^+ momentum from the nucleons, scalar mesons, and vector mesons for ^{16}O , ^{40}Ca , and ^{80}Zr , as well as the nuclear matter limit. In the next section we examine in detail the momentum distributions giving rise to these expectation values.

VI. PLUS-MOMENTUM DISTRIBUTIONS AND LEPTON-NUCLEUS DEEP INELASTIC SCATTERING

We discuss the probability that a nucleon, or meson has a momentum p^+ . In the light-front formulation, these distribution functions are determined by the absolute square of the ground state wave function. Each distribution is discussed in turn.

A. Nucleon plus-momentum distribution

The light-front formulation is very useful for obtaining this observable. The probability that we want, $f_N(p^+)$, follows from Eq. (4.9) as

$$f_N(p^+) = 2 \sum_{\alpha}^{\text{occ}} \int d^2x_{\perp} \left| \langle p^+, \mathbf{x}_{\perp} \mid \psi_{\alpha}^+ \rangle \right|^2, \quad (6.1)$$

with

$$A = \int_0^{\infty} dp^+ f_N(p^+), \quad (6.2)$$

$$P_N^+ = \int_0^{\infty} dp^+ p^+ f_N(p^+). \quad (6.3)$$

The next step is to define a dimensionless variable y :

$$y \equiv p^+ \frac{A}{M_A} \equiv \frac{p^+}{\bar{M}_A}, \quad (6.4)$$

and a dimensionless distribution $f_N(y)$:

$$f_N(y) \equiv \frac{f_N(p^+)}{\bar{M}_A}. \quad (6.5)$$

The result is shown in Fig. 1 for ^{16}O , ^{40}Ca , and ^{80}Zr . The peaks of the distributions range from $y \approx 0.72$ to $y \approx 0.80$, whereas the average values $\langle y \rangle$ are somewhat lower (see Table III). The distribution is not symmetric about its average value, as it would be if a simple Fermi gas model were used. Both of these effects are caused by the presence of nuclear mesons, which carry the remainder of the plus-momentum.

FIGURES

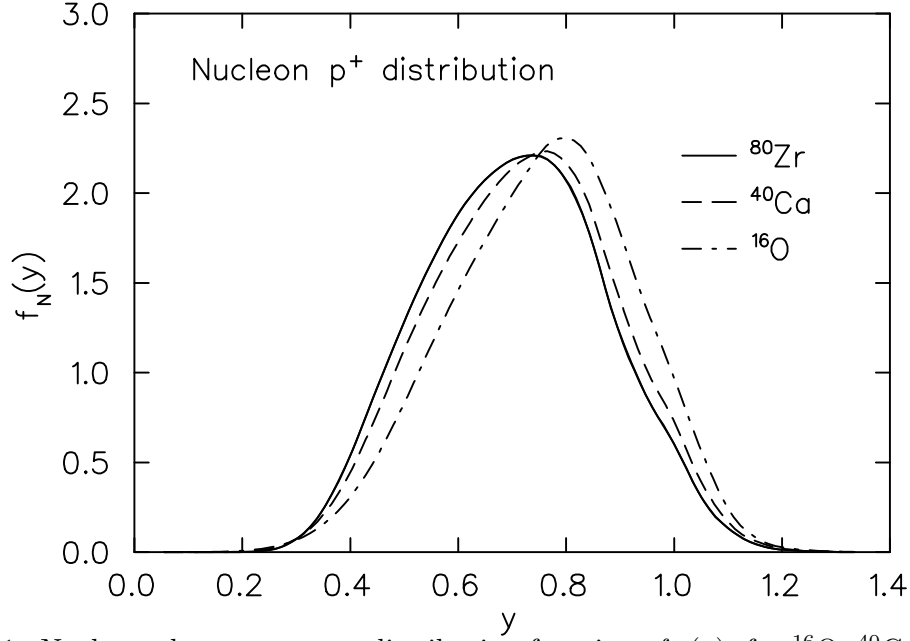


FIG. 1. Nucleon plus-momentum distribution function, $f_N(y)$, for ^{16}O , ^{40}Ca , and ^{80}Zr . Here $y \equiv p^+/(M_A/A)$.

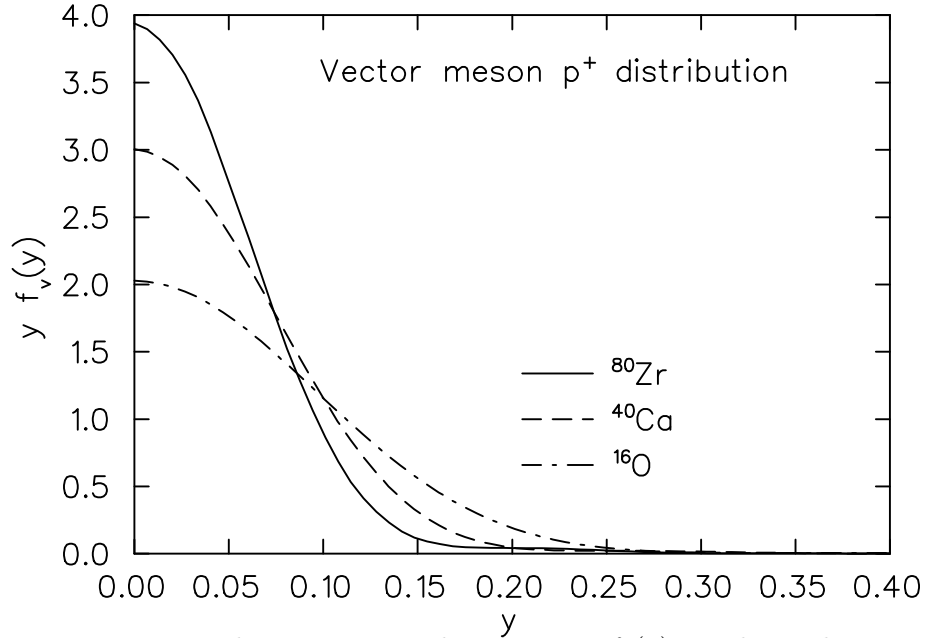


FIG. 2. Vector meson plus-momentum distribution $y f_v(y)$. In the nuclear matter limit, $y f_v(y)$ becomes a delta function.

B. Scalar meson distribution

The probability we want is given by

$$f_s(k^+) = \int d^2k_\perp \langle \Psi | a^\dagger(\mathbf{k}) a(\mathbf{k}) | \Psi \rangle. \quad (6.6)$$

Using Eqs. (3.34) and (3.10) this becomes

$$f_s(k^+) = \int \frac{d^2k_\perp}{(2\pi)^3} \frac{2k^+}{(\mathbf{k}^2 + m_s^2)^2} | \langle \Psi | J(\mathbf{k}) | \Psi \rangle |^2. \quad (6.7)$$

This result which is of the same form as in Ref. [22]. A final step is to define a dimensionless distribution $f_s(y)$:

$$f_s(y) \equiv \frac{f_s(k^+)}{M_A}. \quad (6.8)$$

The scalar mesons are found to carry less than 1% of the plus-momentum of the nucleus (Table III), which is negligible.

C. Vector meson distribution

The probability we want is given by

$$f_v(k^+) = \int d^2k_\perp \sum_{\omega=1,3} \langle \Psi | a^\dagger(\mathbf{k}, \omega) a(\mathbf{k}, \omega) | \Psi \rangle. \quad (6.9)$$

Using Eqs. (3.16) and the mean field approximation (3.5) this becomes

$$f_v(k^+) = \int \frac{d^2k_\perp}{(2\pi)^3} \frac{2k^+}{(\mathbf{k}^2 + m_v^2)^2} \sum_{\omega=1,3} | J(\mathbf{k}, \omega) |^2, \quad (6.10)$$

in which

$$J(\mathbf{k}, \omega) = \int d^3x e^{i\mathbf{k} \cdot \mathbf{x}} \langle \Psi | \bar{\psi}(\mathbf{x}) \gamma \psi(\mathbf{x}) \cdot \bar{\epsilon}(\mathbf{k}, \omega) | \Psi \rangle. \quad (6.11)$$

Using Eq. (2.18) and that only the $\mu = \nu = -$ term enters, leads to the result that

$$f_v(k^+) = \int \frac{d^2k_\perp}{(2\pi)^3} \frac{2k^+}{(\mathbf{k}^2 + m_v^2)^2} \frac{k_\perp^2 + m_v^2}{k^+{}^2} | J_v(\mathbf{k}) |^2, \quad (6.12)$$

another result which is of the same form as in Ref. [22]. A final step is to define a dimensionless distribution $f_v(y)$

$$f_v(y) \equiv \frac{f_v(k^+)}{M}. \quad (6.13)$$

The vector mesons carry approximately 30% of the nuclear plus-momentum. The technical reason for the difference with the scalar mesons is that the evaluation of $a^\dagger(\mathbf{k}, \omega)a(\mathbf{k}, \omega)$ counts vector mesons “in the air” and the resulting expression contains polarization vectors that give a factor of $\frac{1}{k^+}$ in Eq. (6.12) which enhances the distribution of vector mesons of low k^+ . The results for the vector meson distribution are shown in Fig. 2. Clearly as the size of the nucleus increases the enhancement of the distribution at lower values of k^+ becomes more evident. In the case of nuclear matter the distribution $k^+ f_v(k^+)$ becomes a delta function.

D. Lepton-nucleus deep inelastic scattering

It is worthwhile to see how the present results are related to lepton-nucleus deep inelastic scattering experiments. We find that the nucleons carry only about 70% of the plus-momentum. The use of our f_N in standard convolution formulae lead to a reduction in the nuclear structure function that is far too large ($\sim 95\%$ is needed [4]) to account for the reduction observed [4] in the vicinity of $x \sim 0.5$. The reason for this is that the quantity $M + g_s \phi$ acts as a nucleon effective mass of about 670 MeV, which is very small. A similar difficulty occurs in the (e, e') reaction [25] when the mean field theory is used for the initial and final states. The use of a small effective mass and a large vector potential enables a simple reproduction of the nuclear spin orbit force [12,24]. However, effects beyond the mean field may lead to a significant effective tensor coupling of the isoscalar vector meson [26] and to an increased value of the effective mass. Such effects are incorporated in Bruckner theory, and a light-front version [27] could be applied to finite nuclei with better success in reproducing the data.

VII. SUMMARY AND DISCUSSION

The previous Sections present a derivation of a light-front version of mean field theory. The necessary technique is to minimize expectation value of the sum $P^- + P^+$. This leads to a new set of coupled equations (3.31) and (3.30) for the single nucleon modes. These depend on the meson fields of Eqs. (3.13) and (3.21).

The most qualitatively startling feature emerging from the derivation is that the meson field equations (3.13) and (3.21) are the same as that of the usual theory, except that z of the equal-time theory translates to $-x^-/2$ of the light-front version. This can be understood in a simple manner by noting that light-front quantization occurs at $x^+ = 0$. If one then sets $z = -t$, then $x^- = t - z = -2z$. However, this simple argument is not really justified, because using x^\pm precludes the use of z and t . A general argument, using the feature that a static source in the usual coordinates corresponds to a source moving with a constant velocity in light front coordinates, will be presented in a separate paper [15]. That paper also contains a number of solutions of toy models.

Even though the meson field equations of the light-front and equal-time theories are the same, there are substantial and significant differences between the two theories. In our treatment, the mesonic fields are treated as quantum field operators. The mean field approximation is developed by replacing these operators by their expectation values in the complete

ground state nuclear wave function. This means that the ground state wave function contains Fock terms with mesonic degrees of freedom. We can therefore compute expectation values other than that of the field. In particular, we are able to obtain the mesonic momentum distributions (Sec. VI). This feature has been absent in standard approaches.

We obtain an approximate solution (3.48) of our nucleon mode equation. Our nucleon mode functions are approximately a phase factor times the usual equal-time mode functions (evaluated at $x^- = -2z$). This shows that the energy eigenvalues of the two theories should have very similar values. But the wave functions are different— the presence of the phase factor explicitly shows that the nucleons give up substantial amounts of plus momentum to the vector mesons.

A new numerical technique, discussed in Sec. IV and App. A, is introduced to solve the coupled nucleon and meson field equations. Our results display the expected $2j_\alpha + 1$ degeneracy of the single nucleons levels, and the resulting binding energies are essentially the same as for the usual equal-time formulation. This indicates that the approximation (3.48) is valid.

As discussed in Sec. VI-D, the present results related to lepton-nucleus deep inelastic scattering experiments and (e, e') reactions are not consistent with experimental findings. This is because, in ^{40}Ca for example, the nucleons carry only 72% of the plus momentum. This is a result of the quantity $M + g_s\phi$, which acts as a nucleon effective mass, is very small, about 670 MeV. The use of a small effective mass and a large vector potential enables a simple reproduction of the nuclear spin orbit force [12,24]. However, effects beyond the mean field may lead to a significant effective tensor coupling of the isoscalar vector meson [26] and to an increased value of the effective mass. Such effects are incorporated in Bruckner theory [27] which, for infinite nuclear matter, results in nucleons having about 80-85% of the nuclear plus-momentum. A light-front version [27] should be applied to finite nuclei with better success in reproducing the data. Another approach could be to use different Lagrangians, with non-linear couplings between scalar mesons and the nucleons [12], or ones in which the coupling is of derivative form [28]: $\bar{\psi}\gamma^\mu\psi\partial_\mu\phi$. These models are known to have significantly smaller magnitudes of the scalar and vector potentials. In particular, in nuclear matter vector mesons carry only about 10-15% of the nuclear-plus momentum. Another interesting possibility would be to obtain a light-front version of the quark-meson coupling model [29], in which confined quarks interact by exchanging mesons with quarks in other nucleons. This model, also has smaller magnitudes of the scalar and vector potentials.

In any case, these kinds of nuclear physics calculations can be done in a manner in which modern nuclear dynamics is respected, boost invariance in the z -direction is preserved, and in which the rotational invariance so necessary to understanding the basic features of nuclei is maintained.

ACKNOWLEDGMENTS

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APPENDIX A: NUMERICAL TECHNIQUES

There are many possible numerical approaches to solving Eqs. (4.13). We choose a method that is robust, and emphasizes the physical content of the wavefunctions, at the expense of being computationally intensive. We begin by making a Fourier expansion of the wavefunctions in the variable x^- :

$$u_\alpha^\pm(x^-, r) = \frac{1}{\sqrt{2L} 2\pi} \sum_n e^{-ip_n^+ x^-/2} u_{\alpha,n}^\pm(r), \quad (\text{A1})$$

with a similar expansion for $l_\alpha^\pm(x^-, r)$. Boundary conditions are imposed by constraining the system to be in a “box” of a given length in the variable x^- . In the nuclear rest frame, $x^- = -2z$, and so $\partial^+ = 2\partial_- = -\partial/\partial z$. Hence for $-L \leq z \leq +L$, we write

$$u_\alpha^\pm(z, r) = \frac{1}{\sqrt{2L} 2\pi} \sum_n e^{ip_n^+ z} u_{\alpha,n}^\pm(r), \quad (\text{A2})$$

with $\{p_n^+ = nq, n = 1, 2, 3, \dots\}$, and $q = \pi/L$.

For bound states, the functions $u_{\alpha,n}^\pm(r)$ and $l_{\alpha,n}^\pm(r)$ are real, and so the scalar and vector densities take the form

$$\rho^s(z, r) = \sum_{m \geq 0} \rho_m^s(r) \cos mqz, \quad (\text{A3})$$

$$\rho^\pm(z, r) = \sum_{m \geq 0} \rho_m^\pm(r) \cos mqz, \quad (\text{A4})$$

with $m = 0, 1, 2, \dots$, and

$$\begin{aligned} \rho_m^s(r) = \frac{2}{2L 2\pi} \frac{1}{1 + \delta_{m,0}} \sum_\alpha \sum_n^{\text{occ}} & \left[u_{\alpha,n}^+(r) u_{\alpha,n+m}^-(r) + u_{\alpha,n}^-(r) u_{\alpha,n+m}^+(r) \right. \\ & \left. - l_{\alpha,n}^+(r) l_{\alpha,n+m}^-(r) - l_{\alpha,n}^-(r) l_{\alpha,n+m}^+(r) \right], \end{aligned} \quad (\text{A5})$$

$$\rho_m^\pm(r) = \frac{2}{2L 2\pi} \frac{2}{1 + \delta_{m,0}} \sum_\alpha \sum_n^{\text{occ}} \left[u_{\alpha,n}^\pm(r) u_{\alpha,n+m}^\pm(r) + l_{\alpha,n}^\pm(r) l_{\alpha,n+m}^\pm(r) \right]. \quad (\text{A6})$$

The normalization integral for a nucleus with A nucleons is

$$\begin{aligned} A &= 2\pi \int_0^\infty dr r \int_{-L}^L dz \rho^+(z, r) \\ &= 2\pi 2L \int_0^\infty dr r \rho_0^+(r) \\ &= 2 \int_0^\infty dr r \sum_\alpha \sum_n^{\text{occ}} \left[(u_{\alpha,n}^+(r))^2 + (l_{\alpha,n}^+(r))^2 \right]. \end{aligned} \quad (\text{A7})$$

1. Meson fields

The equations for the meson fields are solved using Green function methods. We illustrate this for the vector field V^+ , with results for ϕ following by analogy. Starting with

$$\left(-\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + m_v^2\right) V^+(z, r) = g_v \rho^+(z, r), \quad (\text{A8})$$

we expand $V^+(z, r)$ in the same form as the density $\rho^+(z, r)$, Eq. (A4):

$$V^+(z, r) = \sum_m V_m^+(r) \cos mqz. \quad (\text{A9})$$

The functions $V_m^+(r)$ satisfy

$$\left(-\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + m_v^2 + m^2 q^2\right) V_m^+(r) = g_v \rho_m^+(r), \quad (\text{A10})$$

and their solution may be written as

$$V_m^+(r) = g_v \int_0^\infty dr' r' G(r, r') \rho_m^+(r'). \quad (\text{A11})$$

The Green function is

$$G(r, r') = I_0(m_v^* r) K_0(m_v^* r') \theta(r' - r) + I_0(m_v^* r') K_0(m_v^* r) \theta(r - r'). \quad (\text{A12})$$

We have introduced the definition $m_v^* \equiv \sqrt{m_v^2 + m^2 q^2}$, and I_0 and K_0 are modified cylindrical Bessel functions of zeroth order. The meson fields are computed numerically from Eq. (A11) by an outward and an inward integration.

2. Solution of nucleon equation

To streamline the notation, we drop the explicit dependence on the single particle label α in this section. Equation (4.13) can be rewritten in the form of a 2×2 matrix equation

$$p^- \begin{pmatrix} \langle z, r | u^+ \rangle \\ \langle z, r | l^+ \rangle \end{pmatrix} = \left\{ \left(2g_v V^+ - i \frac{\partial}{\partial z} \right) I + \mathcal{H} \frac{1}{\left(-i \frac{\partial}{\partial z} \right) I} \mathcal{H} \right\} \begin{pmatrix} \langle z, r | u^+ \rangle \\ \langle z, r | l^+ \rangle \end{pmatrix}, \quad (\text{A13})$$

with the constrained subsidiary relation

$$\begin{pmatrix} \langle z, r | u^- \rangle \\ \langle z, r | l^- \rangle \end{pmatrix} = \frac{1}{\left(-i \frac{\partial}{\partial z} \right) I} \mathcal{H} \begin{pmatrix} \langle z, r | u^+ \rangle \\ \langle z, r | l^+ \rangle \end{pmatrix}. \quad (\text{A14})$$

Here I is the 2×2 identity matrix, and

$$\mathcal{H} = \begin{pmatrix} M^* & D_1 + ig_v \frac{\partial \Lambda}{\partial r} \\ D_2 - ig_v \frac{\partial \Lambda}{\partial r} & -M^* \end{pmatrix}, \quad (\text{A15})$$

$$D_1 = - \left(\frac{\partial}{\partial r} + \frac{j_z + \frac{1}{2}}{r} \right), \quad (\text{A16})$$

$$D_2 = \left(\frac{\partial}{\partial r} - \frac{j_z - \frac{1}{2}}{r} \right). \quad (\text{A17})$$

If we take N Fourier components $n = 1, 2, 3, \dots, N$ in the expansion of Eq. (A2) in z , then $u^+(z, r) = \langle z, r | u^+ \rangle$ and $l^+(z, r) = \langle z, r | l^+ \rangle$ have the matrix representation

$$\begin{pmatrix} u_1^+(r) \\ u_2^+(r) \\ \vdots \\ u_N^+(r) \end{pmatrix}, \quad \begin{pmatrix} l_1^+(r) \\ l_2^+(r) \\ \vdots \\ l_N^+(r) \end{pmatrix}, \quad (\text{A18})$$

where $u_n^+(r) = \langle p_n^+, r | u^+ \rangle$ and $l_n^+(r) = \langle p_n^+, r | l^+ \rangle$.

Equation (A13) becomes a $2N \times 2N$ matrix equation. Matrix elements of the $N \times N$ sub-blocks are determined from the integrals

$$[V^+(r)]_{(nn')} = \langle p_n^+ | V^+(z, r) | p_{n'}^+ \rangle \quad (\text{A19})$$

$$= \frac{1}{2L} \int_{-L}^L dz e^{i(p_{n'}^+ - p_n^+)z} V^+(z, r), \quad (\text{A20})$$

$$= \frac{1 + \delta_{m,0}}{2} V_m^+(r) \delta_{|n-n'|,m}. \quad (\text{A21})$$

Similarly,

$$[M^*(r)]_{(nn')} = M \delta_{n,n'} + \frac{1 + \delta_{m,0}}{2} g_s \phi_0(r) \delta_{|n-n'|,m}, \quad (\text{A22})$$

$$\left[-i \frac{\partial}{\partial z} \right]_{(nn')} = p_n^+ \delta_{n,n'}, \quad (\text{A23})$$

$$[D_1]_{(nn')} = D_1 \delta_{n,n'}, \quad (\text{A24})$$

$$[D_2]_{(nn')} = D_2 \delta_{n,n'}, \quad (\text{A25})$$

$$[i\Lambda(r)]_{(nn')} = \frac{-\frac{1}{2}V_m^+(r) + (-1)^m V_0^+(r)}{p_n^+ - p_{n'}^+} \delta_{|n-n'|,m} \quad m \neq 0. \quad (\text{A26})$$

The last relation comes from the definition $-\frac{\partial \Lambda}{\partial z} = V^+$. Using integration by parts, and a careful treatment of surface terms, gives matrix elements in the form (A26).

The problem has now been reduced to an eigenvalue problem involving $2N$ coupled differential equations in the variable r . To solve this, we make a further expansion of $u_n^\pm(r)$ and $l_n^\pm(r)$ in a finite basis of B-splines of degree k [30,31]:

$$u_n^\pm(r) = \sum_{i=1}^{\mathcal{N}} \alpha_{n,i}^\pm B_i^{(k)}(r), \quad (\text{A27})$$

$$l_n^\pm(r) = \sum_{i=1}^{\mathcal{N}} \beta_{n,i}^\pm B_i^{(k)}(r). \quad (\text{A28})$$

The B-splines $\{B_i^{(k)}, i = 1, \dots, \mathcal{N}\}$ are polynomials of degree $k - 1$ spanning a domain of equally spaced knots $\{t_i, i = 1, \dots, \mathcal{N} + k\}$ in r . They are smooth “local” functions that are non-zero only on the interval $t_i < r < t_{i+k}$. This basis forms an accurate, but non-orthogonal set. Hence the overlap matrix

$$S_{ij} = \int_0^\infty dr r B_i^{(k)}(r) B_j^{(k)}(r) \quad (\text{A29})$$

is non-diagonal. It is diagonally banded, however, since $B_i^{(k)}(r)$ and $B_j^{(k)}(r)$ are functions that only overlap if $|i - j| \leq k - 1$. The property of being diagonally banded also applies to the matrix elements of other operators.

We now have a matrix equation with the 2×2 block structure

$$\left\{ \left[2g_v V^+ - i \frac{\partial}{\partial z} \right] \otimes I + \mathcal{H} \frac{1}{\left[-i \frac{\partial}{\partial z} \right] \otimes I} \mathcal{H} \right\} \begin{pmatrix} \alpha^+ \\ \beta^+ \end{pmatrix} = p^- [1] \otimes I \begin{pmatrix} \alpha^+ \\ \beta^+ \end{pmatrix}, \quad (\text{A30})$$

where \otimes denotes an outer product of matrices. $\alpha^+ = \{\alpha_{n,i}^+\}$ and $\beta^+ = \{\beta_{n,i}^+\}$ are column vectors of length $(N \times \mathcal{N})$, and the $(N \times \mathcal{N}) \times (N \times \mathcal{N})$ sub-blocks have matrix elements

$$[V^+]_{(nn'),(ij)} = \int_0^\infty dr r B_i^{(k)}(r) B_j^{(k)}(r) [V^+(r)]_{(nn')}, \quad (\text{A31})$$

$$[M^*]_{(nn'),(ij)} = \int_0^\infty dr r B_i^{(k)}(r) B_j^{(k)}(r) [M^*(r)]_{(nn')}, \quad (\text{A32})$$

$$\left[-i \frac{\partial}{\partial z} \right]_{(nn'),(ij)} = p_n^+ \delta_{n,n'} S_{ij}, \quad (\text{A33})$$

$$[1]_{(nn'),(ij)} = \delta_{n,n'} S_{ij}, \quad (\text{A34})$$

$$[D_1]_{(nn'),(ij)} = \delta_{n,n'} \int_0^\infty dr r B_i^{(k)}(r) \left(-\frac{d}{dr} B_j^{(k)}(r) - \frac{j_z + \frac{1}{2}}{r} B_j^{(k)}(r) \right), \quad (\text{A35})$$

$$[D_2]_{(nn'),(ij)} = \delta_{n,n'} \int_0^\infty dr r B_i^{(k)}(r) \left(\frac{d}{dr} B_j^{(k)}(r) - \frac{j_z - \frac{1}{2}}{r} B_j^{(k)}(r) \right), \quad (\text{A36})$$

$$\left[i \frac{\partial \Lambda}{\partial r} \right]_{(nn'),(ij)} = - \int_0^\infty dr \frac{d}{dr} \left(r B_i^{(k)}(r) B_j^{(k)}(r) \right) [i\Lambda(r)]_{(nn')}. \quad (\text{A37})$$

The last relation follows from using integration by parts.

3. Numerical methods

For our numerical calculations we use $L = 12$ fm and $N = 24$ Fourier components, and set the number of terms in the cosine expansion of the densities to be $N/2$, or 12. We choose $k = 5$ for the degree of the B-splines, $\mathcal{N} = 20$ B-splines in the expansion in r , and take $0 < r < L$. The integrals over r are performed using Gaussian integration between knots, which gives exact results for the matrix elements S_{ij} , Eq. (A29).

The matrix eigenvalue problem Eq. (A30) is of the form $Ax = \lambda Bx$, where A and B are real, symmetric matrices. In our problem, A and B are diagonally banded, and there

are efficient EISPACK routines that take advantage of this [32]. Cholesky decomposition is used to efficiently compute the matrix

$$\frac{1}{\left[-i\frac{\partial}{\partial z}\right] \otimes I} \mathcal{H}, \quad (\text{A38})$$

which is needed both to determine u^- and l^- as well as to construct the matrix in Eq. (A30).

APPENDIX B: MOMENTUM DISTRIBUTIONS

In momentum space the meson field equations become

$$\phi(k^+, \mathbf{k}_\perp) = -\frac{g_s}{\mathbf{k}^2 + m_s^2} \rho_s(k^+, \mathbf{k}_\perp), \quad (\text{B1})$$

$$V^+(k^+, \mathbf{k}_\perp) = \frac{g_v}{\mathbf{k}^2 + m_s^2} \rho^+(k^+, \mathbf{k}_\perp), \quad (\text{B2})$$

with $\mathbf{k}^2 = (k^+)^2 + \mathbf{k}_\perp^2$, and the convention

$$V^+(k^+, \mathbf{k}_\perp) = \int d^2 x_\perp \int dz e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp - ik^+ z} V^+(z, \mathbf{x}_\perp). \quad (\text{B3})$$

The scalar meson momentum distribution from Eq. (6.7) can be rewritten as

$$f_s(k^+) = \frac{2k^+}{2\pi} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{g_s^2}{(\mathbf{k}^2 + m_s^2)^2} |\rho_s(k^+, \mathbf{k}_\perp)|^2 \quad (\text{B4})$$

$$= \frac{2k^+}{2\pi} \int \frac{d^2 k_\perp}{(2\pi)^2} |\phi(k^+, \mathbf{k}_\perp)|^2 \quad (\text{B5})$$

$$= \frac{2k^+}{2\pi} \int d^2 x_\perp |\phi(k^+, \mathbf{x}_\perp)|^2. \quad (\text{B6})$$

Then

$$P_s^+ = \int_0^\infty dk^+ k^+ f_s(k^+) \quad (\text{B7})$$

$$= \frac{1}{2} \int_{-\infty}^\infty \frac{dk^+}{2\pi} 2(k^+)^2 \int d^2 x_\perp |\phi(k^+, \mathbf{x}_\perp)|^2 \quad (\text{B8})$$

$$= \int d^3 x (\partial^+ \phi(\mathbf{x}))^2 \quad (\text{B9})$$

$$= \langle T_s^{++} \rangle. \quad (\text{B10})$$

The vector meson momentum distribution is a little more complicated. Starting from Eq. (6.12),

$$f_v(k^+) = \frac{2k^+}{2\pi} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{g_v^2}{(\mathbf{k}^2 + m_v^2)^2} \frac{k_\perp^2 + m_v^2}{(k^+)^2} |\rho^+(k^+, \mathbf{k}_\perp)|^2 \quad (\text{B11})$$

$$= \frac{2k^+}{2\pi} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{g_v}{\mathbf{k}^2 + m_v^2} \frac{\mathbf{k}^2 + m_v^2 - (k^+)^2}{(k^+)^2} V^+(k^+, \mathbf{k}_\perp) \rho^+(k^+, \mathbf{k}_\perp) \quad (\text{B12})$$

$$= \frac{2}{2\pi} \int \frac{d^2 k_\perp}{(2\pi)^2} \left[\frac{1}{k^+} g_v V^+(k^+, \mathbf{k}_\perp) \rho^+(k^+, \mathbf{k}_\perp) - k^+ |V^+(k^+, \mathbf{k}_\perp)|^2 \right]. \quad (\text{B13})$$

Then

$$P_v^+ = \int_0^\infty dk^+ k^+ f_v(k^+) \quad (\text{B14})$$

$$= \frac{1}{2} \int_{-\infty}^\infty \frac{dk^+}{2\pi} 2 \int d^2x_\perp \left[g_v V^+(k^+, \mathbf{x}_\perp) \rho^+(k^+, \mathbf{x}_\perp) - (k^+)^2 |V^+(k^+, \mathbf{x}_\perp)|^2 \right] \quad (\text{B15})$$

$$= \int d^3x \left[g_v V^+(\mathbf{x}) \rho^+(\mathbf{x}) - (\partial^+ V^+(\mathbf{x}))^2 \right] \quad (\text{B16})$$

$$= \langle T_v^{++} \rangle. \quad (\text{B17})$$

Clearly $f_v(k^+)$ is singular at $k^+ = 0$, so we plot $k^+ f_v(k^+)$ instead.

Momentum distributions involve integrals over \mathbf{x}_\perp , or equivalently over \mathbf{k}_\perp , so one really only needs to Fourier transform $V^+(z, \mathbf{x}_\perp)$ in z . If we define

$$V^+(k^+, \mathbf{x}_\perp) = \int dz e^{-ik^+z} V^+(z, \mathbf{x}_\perp), \quad (\text{B18})$$

then for $k_m^+ = mq$, $q = \pi/L$, it follows from the definition Eq. (A9) that

$$V^+(k_m^+, \mathbf{x}_\perp) = 2L \frac{1 + \delta_{m,0}}{2} V_m^+(r), \quad (\text{B19})$$

with a similar result for $\rho^+(k_m^+, \mathbf{x}_\perp)$, $\phi(k_m^+, \mathbf{x}_\perp)$, and $\rho_s(k_m^+, \mathbf{x}_\perp)$. Hence we calculate the momentum distributions from the expressions

$$k_m^+ f_s(k_m^+) = \frac{2(mq)^2}{q} \frac{(1 + \delta_{m,0})^2}{4} 2\pi 2L \int_0^\infty dr r (\phi_m(r))^2 \quad (\text{B20})$$

$$k_m^+ f_v(k_m^+) = \frac{2}{q} \frac{(1 + \delta_{m,0})^2}{4} 2\pi 2L \int_0^\infty dr r \left[g_v V_m^+(r) \phi_m^+(r) - (mq)^2 (V_m^+(r))^2 \right]. \quad (\text{B21})$$

The nucleon momentum distribution for $p_n^+ = nq$ is given by

$$f_N(p_n^+) = \frac{2}{q} \int_0^\infty dr r \sum_{\alpha}^{\text{occ}} \left[(u_{\alpha,n}^+(r))^2 + (l_{\alpha,n}^+(r))^2 \right], \quad (\text{B22})$$

so that $A = \int_0^\infty dp^+ f_N(p_n^+) \approx q \sum_n f_N(p_n^+)$, and $P_N^+ = \int_0^\infty dp^+ p^+ f_N(p_n^+) \approx q \sum_n p_n^+ f_N(p_n^+)$. We interpolate between the discrete values of k_n^+ and p_n^+ to produce the plots of Figs. 1 and 2.

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